

## MANUFACTURING A GALLON CAN -- A MINIMIZATION PROBLEM

C:\winword\manufacturing a gallon can.doc\0598

A metal can in the shape of a rectangular solid with a square base (top and bottom) is to be manufactured at a minimum cost for materials. It is to hold one gallon of heavy-duty paint remover. The sides are to be made of one gauge of metal that costs \$1.10 per square foot and the top and bottom must be made with a heavier gauge metal that costs \$2.75 per square foot.

Your responsibility is to find the dimensions of the can (to the nearest hundredth of an inch) that minimizes the cost (to the nearest tenth of a penny).

To do so, please answer the following questions. Show all work clearly and explain what you are doing and why. Label your responses with the corresponding letter of the question. Your grade will be based on how well you communicate how you solved the problem, your organizational skills illustrated, as well as accuracy and interpretation of the solution. Enjoy. ☺

- a) Include a sketch of the container in your solution.
- b) Using "sample" dimensions of your choice, find one or two possible set(s) of dimensions that the container could be and how much that container would cost.
- c) Let  $x$  represent the length of the side of the square base. Find an expression in terms of  $x$  for the height of the can. Write your answer in the form of a function:  $h(x) = \dots$
- d) Also letting  $x$  represent the length of the side of the square base, find an expression or formula that computes the total cost of the materials of the can as a function of  $x$ . Write your answer in the form:  $TC(x) = \dots$
- e) Graph the equation you found in part (d) on a graphing utility. Record the window you used.
- f) Graphically find the dimensions of the can that make the cost of the can a minimum. What is that minimum cost?
- g) Check that these dimensions do, in fact, make up a can with the necessary capacity. How far off is it and in which direction? Is this acceptable?
- h) Using construction paper (cardboard, tape, ...) construct a model of this can labeling the dimensions on the model and include the minimum cost to make it also. Print your name, period, and problem number on the model clearly.
- i) Make an appointment to meet with me some time either before school (7:30 – 7:50) or after school (2:30 – 3:00) to check the solution to your problem. Also we will investigate how this problem can be solved using calculus and the TI-92 with the equation you generated.
- j) Write two or three paragraphs that indicate what you learned by doing this problem.

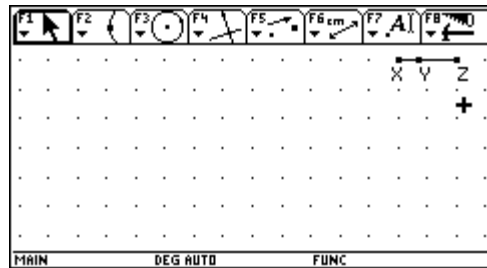
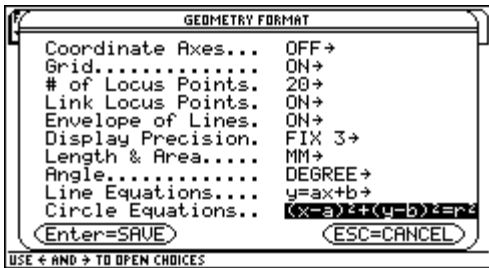
**C 1998 Reardon Gifts, Inc**

# MANUFACTURING A GALLON CAN -- TI-92 Teacher's Notes

C:\winword\mfg gallon can teacher notes.doc\07/23/98

Geometry settings F8 Format as follows:

Using F2 Segment, construct a segment that is two grid spaces in length (10 mm). Using F2 Point on Object, place a point anywhere between.



Here is the plan: We will measure the length of segment XY in mm. We will use that for the length of the side of the square base. The height of the can is dependent on this distance. The following is the mathematical explanation as to how to compute the height:

$$V_{box} = l \cdot w \cdot h$$

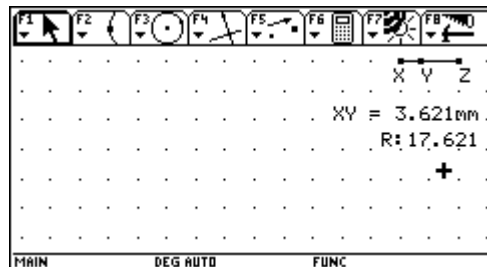
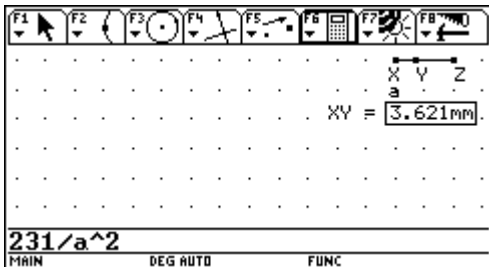
$$231 = x \cdot x \cdot h$$

$$\frac{231}{x^2} = h$$

The volume of the can is to be one gallon, or 231 cubic inches.

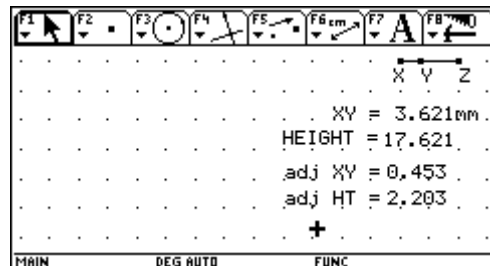
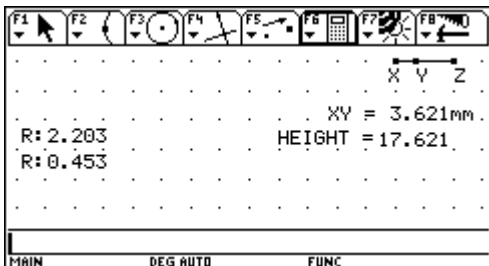
Using F6 Calculate as follows:

Press <Enter> to compute the result. Move that Result as shown. The result is the height of the can.



To make the can's drawing fit on the screen as much as possible we will divide the leg and height each by a scaling factor, in this case, 8. Use F6 Calculate.

These dimensions will be labeled adj XY and adj HT. These numbers will be used only for construction and will be hidden from view later.



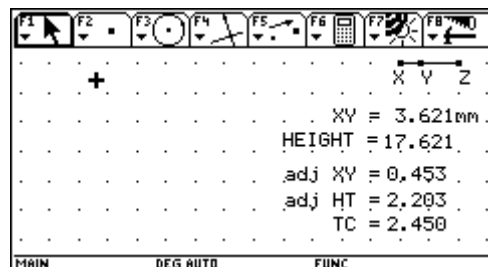
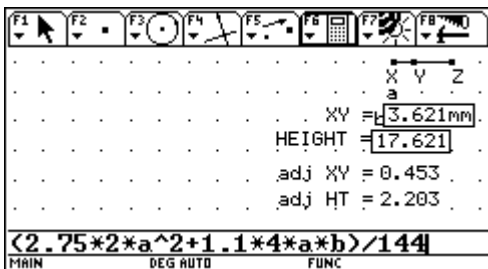
Before we draw a model of this one-gallon can, we will first generate the function that calculates the total cost (TC) of the materials for the can. Let  $x$  represent the length of the side of the square base and  $h$  represent the height of the can. Recall that it is \$2.75 per square foot for the material on the top and bottom and \$1.10 per square foot for the material on the sides. Since we are doing our computations in square inches we have to divide by 144 (square inches in a square foot).

**NOTE:** Do not pass up this teaching moment – most students will claim one square foot is 12 square inches. (*unfortunately*)

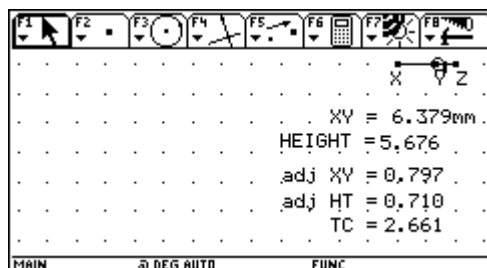
$$TC(x,h) = \frac{2.75 \cdot 2x^2 + 1.10 \cdot 4x \cdot h}{144}$$

The following is how to calculate the TC on the TI-92. The length of the side of the square will be denoted  $a$  and the height will be denoted  $b$ . Use F6 Calculate:

Moving that result over, here is what we have so far: for a length of a base of 3.621 inches and a height of 17.621 inches, the Total Cost of the materials will be \$2.45.



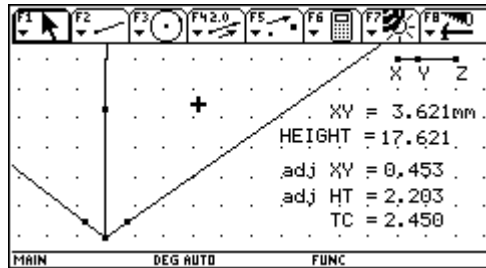
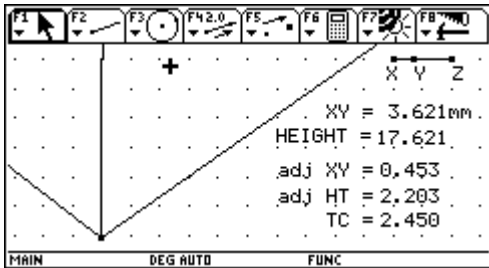
To get other size paint cans and their costs, just “grab” point  $Y$  and slide it along segment  $XZ$ . As you do this, the values of  $XY$ ,  $HEIGHT$ , and  $TC$  are automatically computed. For example:



Now for the cool part. We will draw a model of the can on the TI-92 screen using the values of XY and HEIGHT that will change as we “slide” point Y on segment XZ. We are going to use adj XY and adj HT for our lengths.

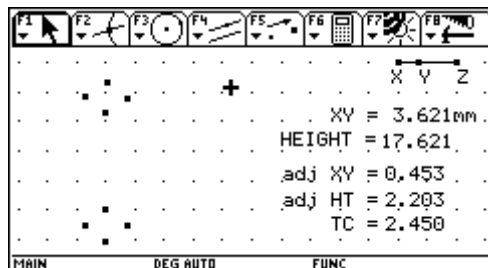
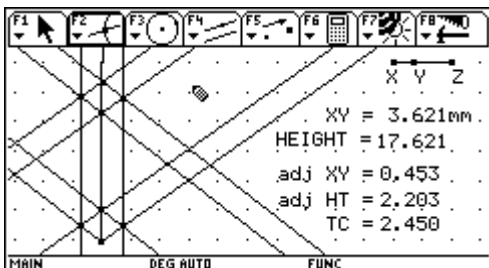
Using F2 Ray we will construct 3 rays with the same vertex as shown. Make sure that the “middle” one is vertical, that is, passes through a grid point. \*\*\*hide the point on the middle ray.

Using F4 Measurement Transfer, transfer the length of adj XY to the two NON-vertical rays and the adj HT to the vertical ray as shown.



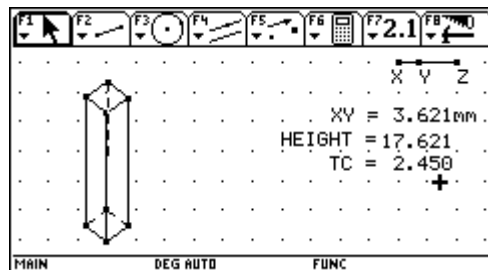
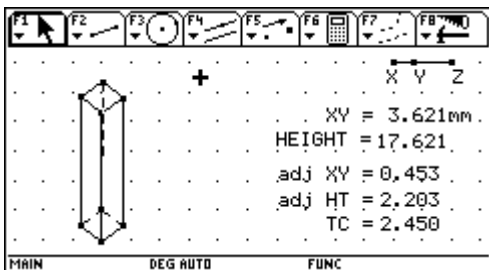
To create the illusion of a paint can, construct parallel lines using F4 Parallel Line and indicate the points of intersection using F2 Intersection Point as shown.

Now this looks confusing, so let's hide the lines and rays using F7 Hide / Show. Then we will connect vertices with segments (three of them dotted -- guess which ones).

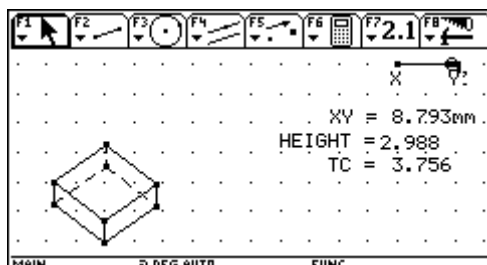
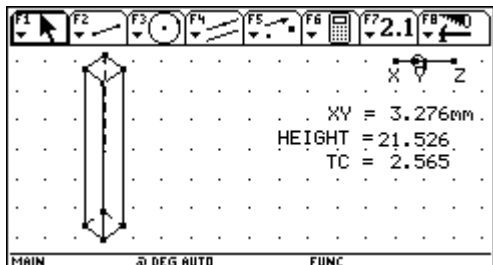


Now complete the drawing of the model as shown below. Use F2 Segment but make 3 of the segments dotted using F7 Dotted.

It is now time to hide the adj XY and the adj HT. They are not necessary to be seen but necessary for the drawing of the model. Use F7 Hide / Show

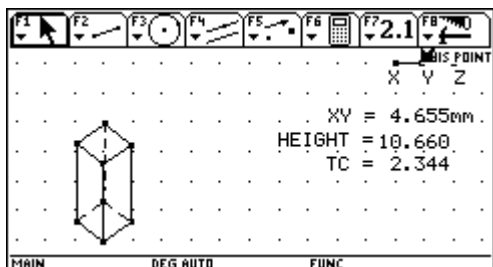


Finally this is the screen that you can use in your class presentation. After explaining the problem to the class, demonstrate the dynamic explanation of this problem by “grabbing” point Y on segment XZ and slowly dragging it. As you do so, explain what each number means and how the paint remover can will look. Try to find the dimensions of the can that yields the lowest cost. Ask students to compare that with what they computed the solution to be graphically. Below are a couple of possible screens:



**NOTE:** Point out why the drawing can go off the screen.

Below is the screen for what I found to be the optimum solution:



It is important that the students know how to interpret this solution. Also they should realize that this is not the most accurate solution – just the best to the precision allowed by the drawing on the TI-92.

### LET’S TAKE THIS FURTHER

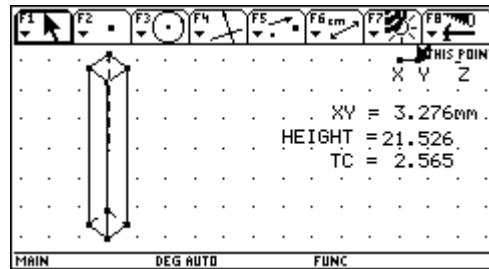
By using the animation feature on the TI-92 and the Data/Matrix Editor, it is possible to gather data from this dynamic model, graph the data and perhaps generate a regression equation that may model this behavior. Oh wow.

First some housekeeping ideas. We are going to store the data generated in a data file called Sysdata. It is good practice (and very necessary) to clear out that file before storing new data to it. To do this, open the VAR-LINK menu (2<sup>nd</sup> -) and delete the file named “sysdata DATA” that is stored in the MAIN folder.



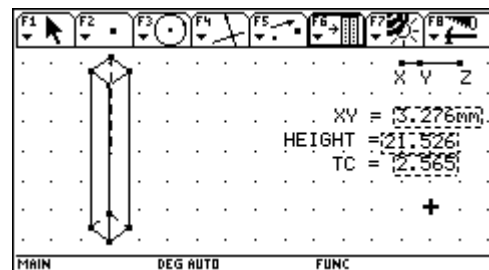
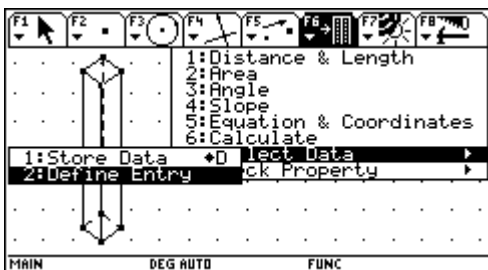
Press the <ESC> key to return to the Geometry screen.

We will not collect data for all values from 0 to 10 inches for the length of the side of the square base, but for “meaningful” values. “Grab” point Y and position it about where I have it in the figure below:



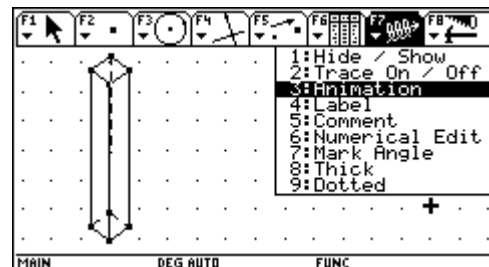
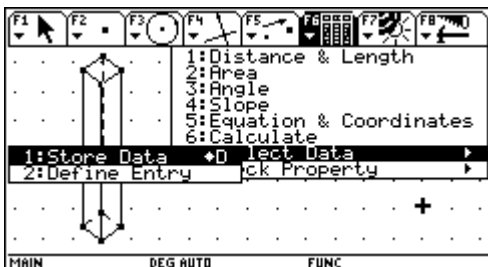
Using F7 Collect Data, we must Define Entry first (even though it is listed as number 2)

Select (highlight or marquee) the numerical values for XY, HEIGHT, and TC, in that order as shown below. These are the values that will be stored in the Data / Matrix Editor.



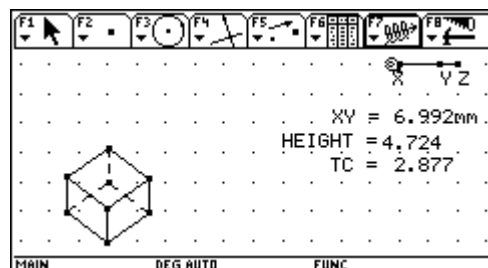
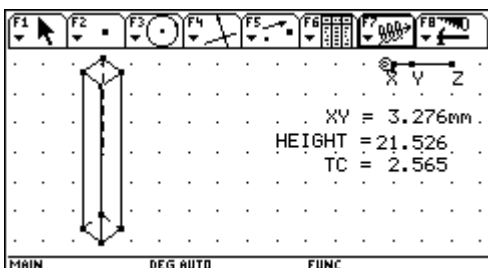
Next use the Store Data command under the F6 Collect Data menu

After pressing <Enter> Press F7 Animation



Move your cursor to point to point Y, “grab” it and pull it to the left to animate it. The data that appears on the screen will be collected in the Data / Matrix Editor.

As the data is being collected, watch the screen and stop the collection when the data seems to be clearly getting away from a minimum cost. Terminate the animation and data collection by pressing <ENTER>.



To view the data press the <APPS> key and choose Data / Matrix Editor, Current. Notice that I typed side for c1, height for c2, and cost for c3.

	side	height	cost		
	c1	c2	c3	c4	c5
1	3.2759	21.526	2.5645		
2	3.3835	20.179	2.5234		
3	3.4586	19.311	2.4977		
4	3.5338	18.498	2.4743		
5	3.609	17.735	2.4532		
6	3.6842	17.019	2.4343		
7	3.7594	16.345	2.4173		

c1=

MAIN RAD AUTO FUNC

Now we will plot the data in an attempt to model and analyze it. Press F2 Plot Setup, then F1 define to get to this screen. Fill in the options as listed below.

sysdata Plot 1	
Plot Type.....	Scatter→
Mark.....	Box→
X.....	c1
Y.....	c3
Higl. Guplet Width	1
Use Freq and Categories?	NO→
Pres.....	
Categori.....	
(inc)ude Categori.....	C
Enter=SAVE ESC=CANCEL	

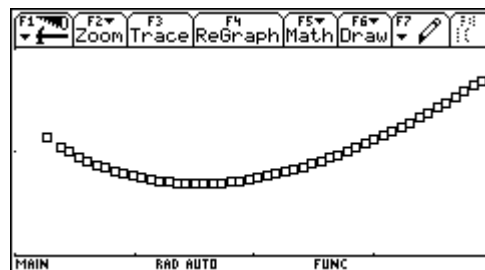
USE ← AND → TO OPEN CHOICES

Choose an appropriate window

Zoom	
xmin=	3.
xmax=	7.
xsc1=	1.
ymin=	2.
ymax=	3.
ysc1=	5
xres=	24

MAIN RAD AUTO FUNC

Then graph side vs. total cost:

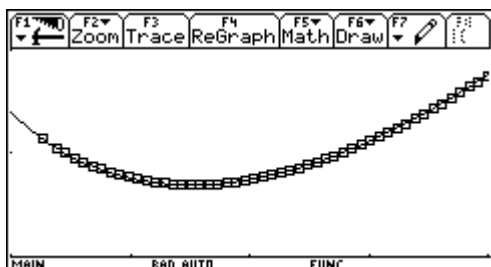


It is here that you or the students may want to engage in trying to find a curve of best fit using the regression features on the calculator. However I think it would be more instructive and **more powerful** if we typed the equation that the students were to generate earlier for the total cost in terms of  $x$ ,  $x$  being the length of the side of the base. Type this into  $y1(x)$  as shown below:

Edit	
Plot 1:	$2.75 \cdot 2 \cdot x^2 / 144 + 1.1 \cdot 4 \cdot x \cdot 231$
y1=	
y2=	
y3=	
y4=	
y5=	
y6=	
y1(x)=2.75*2*x^2/144+1.1*4*x*...	

MAIN RAD AUTO FUNC

Now for the OOHS and the AAHS. Graph that equation to see just how good a “fit” it is.



Now discuss / summarize what we did and what it means.