12 Days of Christmas Functions

During this holiday season we hear often the “12 Days of Christmas” song. Of course I see mathematics in this song – lots of wonderful mathematics – joyous mathematics, mathematics of the season.

Please show all work, answers, and a hand sketch (with units shown on the axes) of the graph in the space provided for each.

1. What I would like you to do is to come up with two functions. The first, call it g(d), where d is the day number of the song, and g is the number of gifts sung about on that day. For example, g(5) would be equal to 15 which is 1 + 2 + 3 + 4 + 5. Have the formula written as a single fraction in factored form also. Store it in y1.

2. The second function, call it t(d), where d is the day number of the song, and t is the total number of gifts sung about after that day. For example, t(5) would be equal to 35, which is 1 + 3 + 6 + 10 + 15. Have that formula written as a single fraction in factored form also. Store it in y2.

3. Using the appropriate formula you generated, answer the following questions:

A. How many gifts were sung about on the 10^{th} day? Show or discuss how you found your answer.

B. What is the total number of gifts sung about after the 10^{th} day? Show or discuss how you found your answer.

C. How many gifts were sung about on the 12^{th} day? Show or discuss how you found your answer.

D. What is the total number of gifts sung about by the song's end? Show or discuss how you found your answer.

As always show all work, clearly indicating how you arrived at your functions and solutions in order to receive full credit – COMMUNICATE!!! Leave your equations, graphs, plots, in your calculator for me to see.

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* "inspired" by Mary Lopushansky, Houston, Texas

12 Days of Christmas Functions -- Teachers Notes & Solutions

1. If you use a quadratic regression you will get an exact equation to model this data.

$$y1 = .5x^2 + .5x$$

$$g(d) = .5d^2 + .5d = \frac{d(d+1)}{2}$$ (in factored form)

Make sure that students do include a hand sketch of this graph with the units shown on the axes. The graph is not shown here but left to the reader.

2. If you use a cubic regression you will get an exact equation to model this data.

$$y2 = .1666666666x^3 + .5x^2 + .333333333x$$

$$t(d) = \frac{1}{6}d^3 + \frac{1}{2}d^2 + \frac{1}{3}d = \frac{d(d+2)(d+1)}{6}$$ (in factored form)

Make sure that students do include a hand sketch of this graph with the units shown on the axes. The graph is not shown here but left to the reader. (Don't you just hate it when people say that?)

3. A) g(10) = y1(10) = 55

OR you could trace on y1 at x = 10
OR you could look in the table

B) t(10) = y2(10) = 220

OR you could trace on y2 at x = 10
OR you could look in the table

C) g(12) = 78 (OR trace or table)

D) t(12) = 364