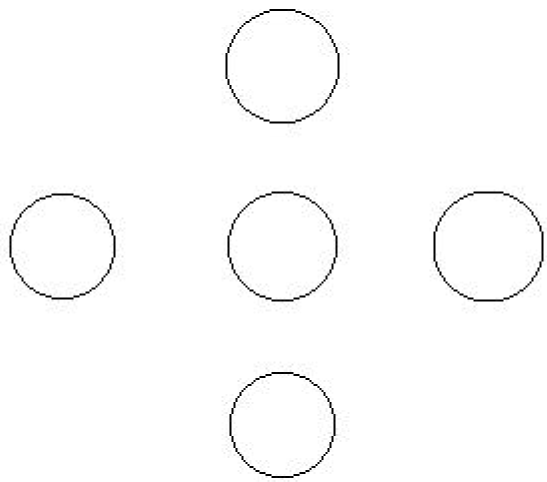
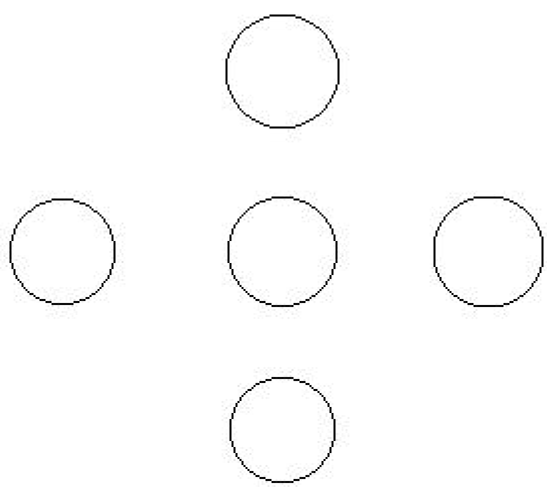
STUDENTS PROBLEMS USING PROBLEM SOLVING STRATEGIES

PSS 1 GUESS AND CHECK

EX. 1 Copy the figure below and place the digits 1, 2, 3, 4, and 5 in these circles so that the sums across (horizontally) and down (vertically) are the same. Is there more than one solution?

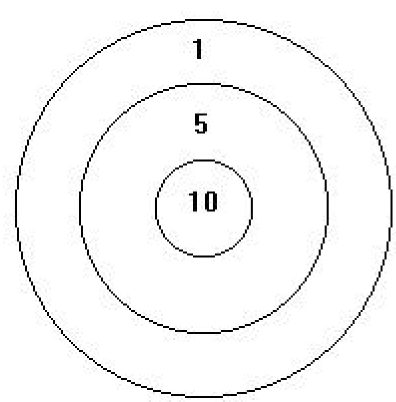


1. Put the numbers 2, 3, 4, 5, and 6 in the circles to make the sum across and the sum down equal to 12. Are other solutions possible? List at least two, if possible.



EX. 2

Three darts hit this dart board and each scores a 1, 5, or 10. The total score is the sum of the scores for the three darts. There could be three 1's, two 1's and 5, one 5 and two 10's, And so on. How many different possible total scores could a person get with three darts?



2. List the 4-digit numbers that can be written using each of 1, 3, 5, and 7 once and only once. Which strategy did you use?

EX. 3 In a stock car race, the first five finishers in some order were a Ford, a Pontiac, a Chevrolet, a Buick, and a Dodge.

- The Ford finished seven seconds before the Chevrolet.
- The Pontiac finished six seconds after the Buick.
- The Dodge finished eight seconds after the Buick.
- The Chevrolet finished two seconds before the Pontiac.

In what order did the cars finish the race? What strategy did you use?

3. Four friends ran a race:

- Matt finished seven seconds ahead of Ziggy.
- Bailey finished three seconds behind Sam.
- Ziggy finished five seconds behind Bailey.

In what order did the friends finish the race?

EX. 4 Pedar Soint has a special package for large groups to attend their amusement park: a flat fee of $20 and $6 per person. If a club has $100 to spend on admission, what is the most number of people who can attend?

4. Stacey had 32 coins in a jar. Some of the coins were nickels, the others were dimes. The total value of the coins was $2.80. Find out how many of each coin there were in the jar. What problem solving strategy did you use?

EX. 5 Continue these numerical sequences. Copy the problem and fill in the next three blanks in each part.

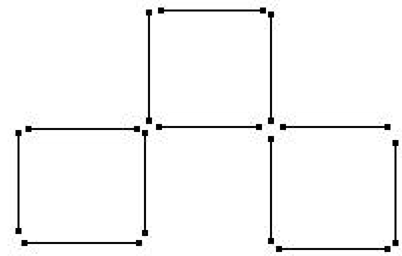
- 1, 4, 7, 10, 13, _____, _____, _____.
- 19, 20, 22, 25, 29, _____, _____ _____.
- 2, 6, 18, 54, _____, _____, ______.

5. Copy and continue the numerical sequences:
a) 3, 6, 9, 12, _____, _____, _____
b) 27, 23, 19, 15, 11, _____, _____, _____
c) 1, 4, 9, 16, 25, _____, _____, _____
d) 2, 3, 5, 7, 11, 13, _____, _____, _____

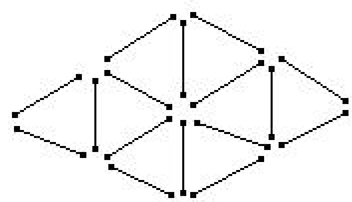
Ex. 6 The houses on Main Street are numbered consecutively from 1 to 150. How many house numbers contain at least one digit 7?

6. The houses on Market Street are numbered consecutively from 1 to 150. How many house numbers contain at least one digit 4?

Ex. 7 The figure below shows twelve toothpicks arranged to form three squares. How can you form five squares by moving only three toothpicks?



7. Sixteen toothpicks are arranged as shown. Remove four toothpicks so that only four congruent triangles remain.



Ex. 8 Suppose that you buy a rare stamp for $15, sell it for $20, buy it back for $25, and finally sell it for $30. How much money did you make or lose in buying and selling this stamp?

8. Suppose that you buy a rare stamp for $15, sell it for $20, buy it back for $22, and finally sell it for $30. How much money did you make or lose in buying and selling this stamp?

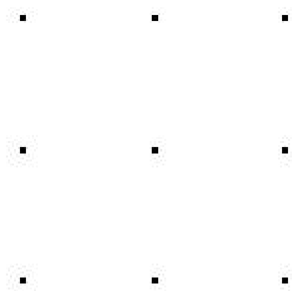
Ex. 9 Ana gave Bill and Clare as much money as each had. Then Bill gave Ana and Clare as much money as each had. Then Clare gave Ana and Bill as much money as each had. Then each of the three people had $24. How much money did each have to begin with?

9. I went into a store and spent half of my money and then $20 more. I went into a second store and spent half of my money and then $20 more. Then I had no money left. How much money did I have when I went into the first store?

Ex. 10 Three apples and two pears cost 78 cents. But two apples and three pears cost 82 cents. What is the total cost of one apple and one pear?

10. Five oranges and a banana cost 87 cents. An orange and five bananas cost 99 cents. What is the total cost of two oranges and two bananas?

Ex. 11 Show how to draw four line segments through the nine dots shown below without lifting your pencil from the paper.



11. You have six sticks of equal length. Without altering the sticks in any way, show how to arrange them end-to-end to form four equilateral triangles.

Ex. 12 Two apples weigh the same as a banana and a cherry. A banana weighs the same as nine cherries. How many cherries weigh the same as one apple?

12. Three pears weigh the same as a quince. A quince weighs as much as eighteen raspberries. How many raspberries weigh the same as a pear?

EXTRA PROBLEMS

Do not write on this paper. Copy the problem as necessary and show all work on YOUR paper. Do not erase wrong tries. Show your thinking!!!
And always state which Problem Solving Strategy(ies) you used.

13. a) This is a magic square that uses each of the digits from 1 to 9, inclusive, because the sum of the numbers in each row, column, and diagonal is the same number, 15.

4	9	2
3	5	7
8	1	6

b) Using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 to create another magic square.

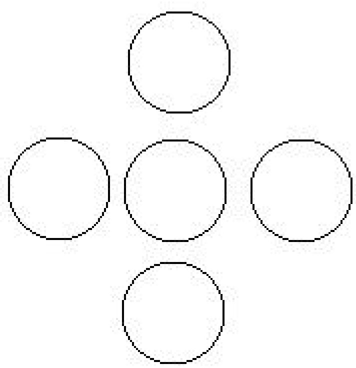
14. a) Using the magic square in problem 13a, interchange the 2 and 8 and the 4 and 6 to create what is called a magic subtraction square. Write down this new square. Add the two end entries and subtract the middle entry from that sum for each row, column, and diagonal. Explain in words what pattern you notice. This is what makes this a magic subtraction square.

14. b) Make a magic subtraction square using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8. Explain in words why this is a magic subtraction square.

15. Jacinski's Hardware has a number of bikes and tricycles for sale. There are 27 seats and 60 wheels all together. Determine how many bikes there are and how many tricycles there are. Please find your answer using at least two different Problem Solving Strategies.

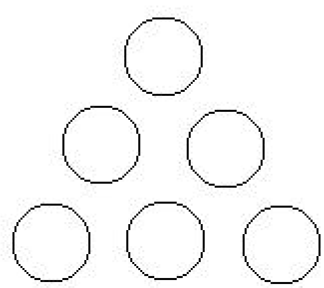
16. Mr. McGlynn has 32 stamps, some are 18-cent stamps, the rest are 27-cent stamps. The stamps are worth $7.65. How many of each stamp does Mr. McGlynn have? Explain how you thought through your solution in a sentence or two.

17. Place the digits 4, 6, 7, 8, and 9 in the circles to make the sum across and vertically equal 19. Is there more than one answer? Explain briefly.

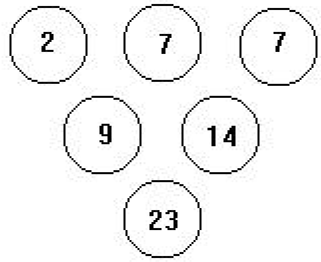


18. Mr. Latessa has nine coins with a total value of 48 cents. What coins does Mr. Latessa have?

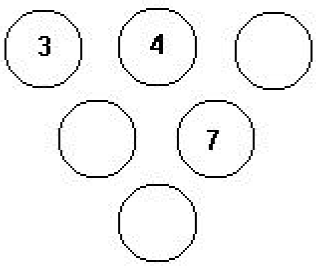
19. Using each of 1, 2, 3, 4, 5, and 6 once and only once, fill in the circles so that the sums of the numbers on each of the three sides of the triangle are equal. Is there more than one solution?



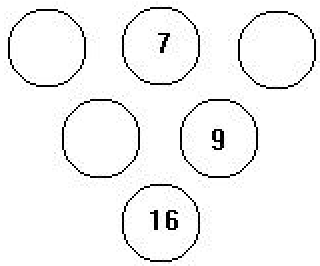
20. In this diagram, the sum of any two horizontally adjacent numbers is the number immediately below and between them. Using the same rule of formation, complete the other arrays.



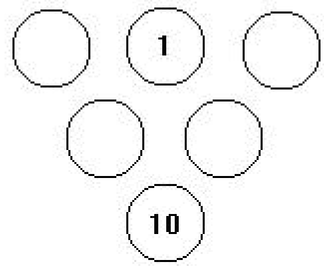
a)



b)



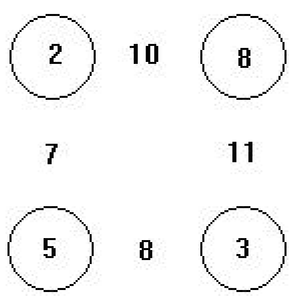
c)



d) Is there more than one solution to parts a or b or c? Explain.

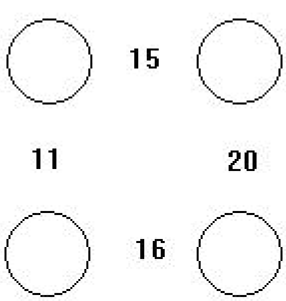
21. Study the sample diagram. Note that:

2 + 8 = 10 5 + 3 = 8 2 + 5 = 7 3 + 8 = 11



If possible, complete each of these diagrams so that the same pattern holds:

a)



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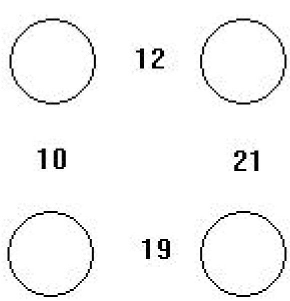
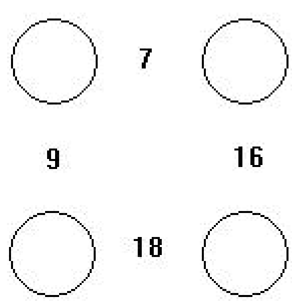
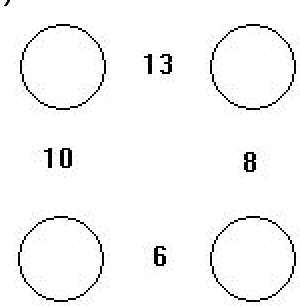


c)

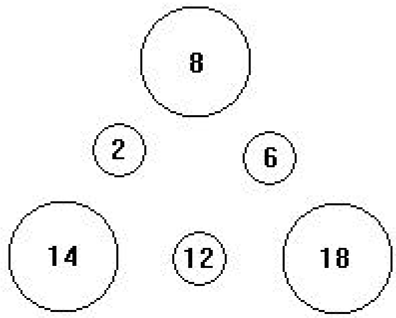


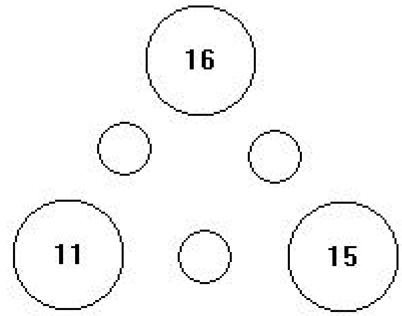
d)



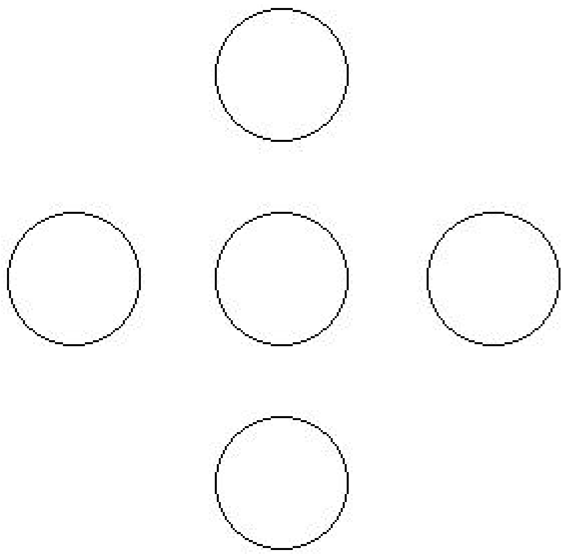
22. Mr. Procopio has nine coins with a total value of 62 cents. What coins does he have?

23. The numbers in the big circles are found by adding the numbers in the two adjacent smaller circles as shown. Complete the second diagram so that the same pattern holds.



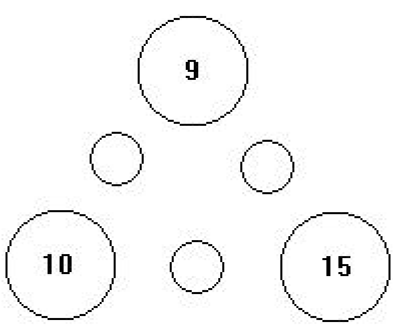


24. Place the numbers 2, 3, 4, 5, and 6 in the circles to make the sum across and the sum down equal 12.

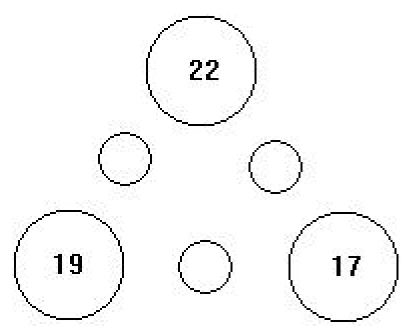


25. The numbers in the big circles are the sums of the numbers in the two small adjacent circles. Place the numbers in the empty circles in each of these arrays so that the scheme holds.

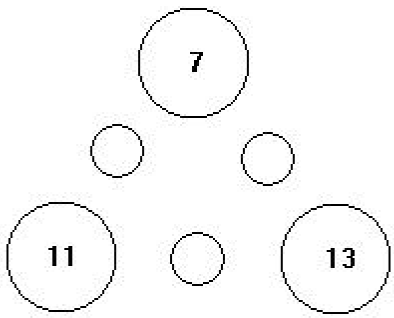
a.



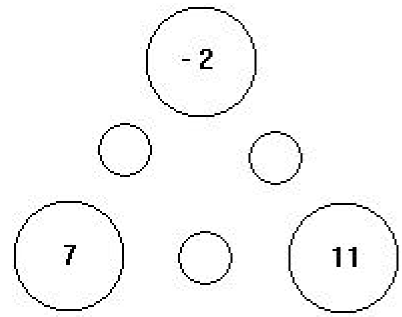
b.



c.



d.



26. How many different amounts of money can you pay if you use three coins including only nickels, dimes, and quarters?

27. List the four-digit numbers that can be written using each of 1, 3, 5, and 7 once and only once.

28. A rectangle has an area of 72 square inches. Its length and width are whole numbers. a) What are the possible dimensions of the rectangle? b) Which of those dimensions yield a rectangle with the smallest perimeter?

29. A rectangle has a perimeter of 28 cm. a) What are the possible dimensions of the rectangle? b) Which of those dimensions yield a rectangle with the largest area?

30. Gertrude and Heathcliffe each worked a different number of days but earned the same amount of money. Use these clues to determine how many days each worked:

Gertrude earned $20 a day.
Heathcliffe earned $30 a day.
Gertrude worked five more days than Heathcliffe.

31. Della can cut through a log in one minute. How long will it take Della to cut a 20 foot log into 2-foot sections?

32. How many posts does it take to support a straight fence 200 yards long if a post is placed every 20 yards?

33. How many posts does it take to support a fence around a square field measuring 200 yards on a side if posts are placed every 20 yards?

34. Carl Friedrich Gauss (1777 – 1855) is generally regarded as one of the three greatest mathematicians of all time. He is sometimes referred to as the "Prince of mathematicians". (Or is he "the mathematician formerly known as Prince"? ☺) Anyway, when just a young school boy, his teacher instructed the students in his class to add all the whole numbers from 1 to 100, inclusive, expecting this to take a long time. To the teacher's astonishment, young Gauss completed the task in about 30 seconds. (Keep in mind that this is long before calculators of any kind!) How did Gauss do this?

35. Find the sum of the whole numbers from 1 to 900, inclusive.

36. Find the sum of the even numbers from 1 to 500, inclusive.

37. Find the sum of the odd numbers from 1 to 700.

38. One of the most interesting and useful patterns in all of mathematics is the numerical array called Pascal's triangle. It is named after the French mathematician Blaise Pascal (1623 – 1662) who showed that these numbers play an important role in the theory of probability. Copy the array and fill in the next three rows of numbers by recognizing the pattern(s) that you notice.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

39. Compute the sum of the elements in each row of Pascal's triangle (shown above). Do this for the first 6 rows. Look for a pattern. See if you can predict the sum of the elements for the seventh row, then check by adding. Do the same for rows 8 and 9. State the general rule for the sum of the elements in words.

40. Copy the following onto your paper. Look for a pattern and fill in the next three blanks with the most likely choices for each sequence. (NOTE: More than one correct answer is possible. Just make sure that you can support your answer with an explanation of the pattern that you observe.)

a) 2, 5, 8, 11, _____, _____, _____

b) 3, 6, 12, 24, 48, _____, _____, _____

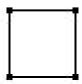
c) 1, 3, 4, 7, 11, 18, _____, _____, _____

d) 4, 9, 16, 25, 36, 49, _____, _____, _____

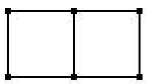
e) 29, 22, 15, 8, _____, _____, _____

f) 3, 9, 27, 81, _____, _____, _____

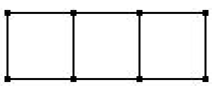
41. a) How many rectangles are there in each of these figures?

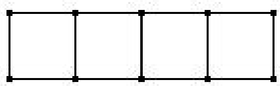


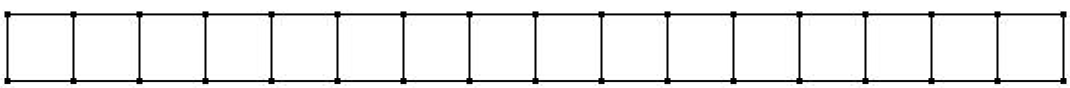
One rectangle



Three rectangles







42. West School has teams only in volleyball, swimming, soccer, and basketball. Erica, Justin, Molly, and Dave each play a different sport. Justin's sport does not use a ball. Molly is older than the volleyball player. Neither Molly nor Dave plays soccer. Who plays what sport?

43. Consider this mathematical sequence of operations:

Input a number
Add 7
Multiply the result by 6
Subtract 18 from the previous result
Divide by 2
Output the number

a) What number would you have to use as input if you wanted 39 as the output?
b) What number would you have to input to obtain an output of 57?

44. Moe, Larry, and Curly are brothers. One day, while in a hurry, they left home with each one wearing the hat and coat of one of the others. Larry was wearing Moe's coat and Hiram's hat. Whose hat and coat was each one wearing?

45. Beth, Jane, and Mitzi play on the basketball team. Their positions area forward, center, and guard. Beth bought a milk shake for Mitzi. Beth is not the forward. Who plays each position?

46. Messy Potaymeeuh bought \$4.79 worth of candy. In how many ways can Messy receive change if she pays with a five dollar bill?

47. Eyecot Aphish is thinking of a number. If you double her number and subtract 7 you obtain 11. What is Eyecot's number?

48. a) Write down the next three rows to continue this sequence of equations.

$1 = 1 = 1^3$

$3 + 5 = 8 = 2^3$

$7 + 9 + 11 = 27 = 3^3$

$13 + 15 + 17 + 19 = 64 = 4^3$

b) Consider the sequence 1, 3, 7, 13, … of the first terms in the sums in part a. Write the first ten terms of this sequence.

49. Write down the next three rows to continue this sequence of equations.

$2 = 1^3 + 1$

$4 + 6 = 2^3 + 2$

$8 + 10 + 12 = 3^3 + 3$

50. Lori Ann Marie wants to know where you will be when you take:

$\frac{3}{5}$ of a CHICK, $\frac{2}{3}$ of a CAT, and $\frac{1}{2}$ of a GOAT ?

51. Suppose you were given \$1,000,000 – but you had to spend it all in one year. How much money would you have to spend per hour to "get rid" of all the money? Round your answer to the nearest penny.

52. How many miniature candy bars can Maximus and Minerva consume in 20 minutes if Maximus can eat two and a half candy bars in 30 seconds and Minerva ate $\frac{3}{4}$ as many candy bars as Maximus?

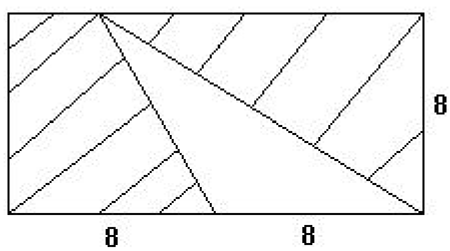
53. Mr. Polya invented a new type of mathematics operation called "duh club", denoted by the symbol, ♣. Mr. Polya told his class that: A ♣ B = $B^2 - 2A$. Find the value of:
(12 ♣ 6) ♣ 13

54. Shirley U. Jest was 3 minutes late to her mathematics class. As a special assignment, her teacher asked her to figure out how many math class periods she would miss in a school year if she was 3 minutes late every day. (NOTE: Class periods are 45 minutes long and there are 180 days in the school year.)

55. If there are exactly four Sundays in December, then December 31st could not fall on exactly three days of the week. Which three days are those?

56. Which of the following is the largest number (No calculators allowed):
2^{100} 3^{75} 5^{50}

57. Find the area of the shaded region in the rectangle shown below:



58. Carl LaFong (capital L, small a, capital F, small o, small n, small g) sold tickets to a school play for $4 each. If the tickets were numbered consecutively from 50 through 84, how much money did Carl LaFong (capital L, small a, capital F, small o, small n, small g) collect?

59. Find the product of: $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)...\left(1-\frac{1}{40}\right)$

60. Find the units digit of the expression 3^{4027}

61. What is the next number in this sequence: 1, 4, 10, 19, 31, … ?

62. The operation "star" is defined as follows: a ∗ b = $a^2 + b$
Find the value of $(-6 * 4) * 8$.

63. A survey of students at Frank Ohl Middle School revealed the following:

240 students are taking mathematics
195 students are taking English
215 students are taking science
150 students are taking mathematics and English
180 students are taking mathematics and science
130 students are taking English and science
100 students are taking all three subjects

How many students are included in the survey?

64. What is the greatest number of 2-inch by 3-inch rectangular cards that can be cut from a rectangular sheet that measures 2 feet by 3 feet?

65. In three bowling games, Lori scored 139, 143, and 144. What score will she need in a fourth game in order to have an average score of 145 for all four games?

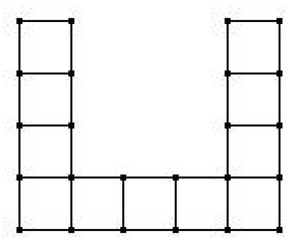
66. Raymond got an 85, 88, and 93 on his first three tests. What must he get on his fourth test so that his average on the four tests is 90 (the lowest 'A')?

67. If today is Tuesday, what day of the week will it be in 100 days from now?

68. A bag of marbles can be divided in equal shares among 2, 3, 4, 5, or 6 friends. What is the least number of marbles that the bag could contain?

69. The four-digit number 3AA1 is divisible by 9. Find the value of A.

70. The figure at the right is formed by eleven squares of the same size. If the area of the figure is 176 sq cm, what is its perimeter?



71. A bag contains 500 beads of the same size. There are 5 different colors of beads and 100 beads of each color. If you are blindfolded, what is the least number of beads that you must pick before you can be sure that you have picked 5 beads of the same color?

72. You are told that the perimeter of a certain rectangle measures 22 inches. If its length and width are each whole numbers of inches, how many different areas (in square inches) are possible for this rectangle?

73. The last Friday of a certain month is the 25th day of the month. What day of the week is the first day of the month?

74. Sue and Ann earned the same amount of money, although one worked 6 more days than the other. If Sue earns $36 a day and Ann earns $60 a day, how many days did each work?

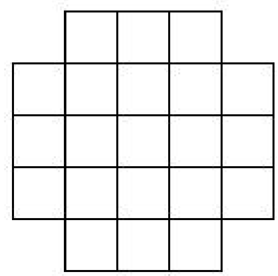
75. A work crew of 3 persons requires 3 weeks and 3 days to do a certain job. How long would it take a work crew of 4 persons to do the same job if they work at the same rate?

76. My favorite History of Mathematics book has 500 pages numbered 1, 2, 3, and so on. How many times does the digit '1' appear in the page numbers? NOTE: The number 141 would have two 1's counted.

77. Suppose that X and Y are two different numbers selected from the first fifty counting numbers (1 to 50 inclusive). What is the greatest possible value of the following expression?

$$\frac{X+Y}{X-Y}$$

78. The average capacity of a set of five containers is 13 liters. A container with a capacity of 7 liters is added to the set of five. What is the average capacity of the set of six containers?



79. Each of the small boxes in the figure at the right is a square. What is the total number of different squares that can be traced using the segments of the figure?

80. Emily spent two thirds of her money. Then she lost two thirds of the money that was left. Four dollars remained. How much money did Emily have to begin with?

81. What day of the week was yesterday if five days before the day after tomorrow was Wednesday?

82. A camera and case together cost $110. If the camera costs $100 more than the case, how much does the case cost?

83. Sarah went to a store, spent half of her money, and then spent $10 more. She went ot a second store, spent half of her remaining money, and then spent $10 more. Then she had no money left. How much money did she have in the beginning when she went to the first store?